Physics Laboratory Guide

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General Information

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Laboratory Manuals:	*Physics Laboratory Manual, Physics I & II (PHYS 2201 & 2202)*
Authors:	Physics Staff
Publisher:	School of Natural Sciences University College Fairleigh Dickinson University
Reference:	*Introduction to Error Analysis: The Study of Uncertainties in Physical Measurements* (Second Edition, 1997)
Author:	John R. Taylor, University of Colorado, Boulder
Publisher:	University Science Books, Sausalito, CA. ISBN 0-935702-75-X
Resources:	http://TheFlorys.org/David.Flory/PhysicsResources.php
Web Site:	http://TheFlorys.org/David.Flory/Physics.Courses.php

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Laboratory Objectives

The overall objectives of a Physics Laboratory are threefold: (1) to demonstrate and make concrete through actual experience some of the physical phenomena presented in the lecture portion of the course; (2) to present in an actual laboratory environment some of the methods and techniques used to investigate physical phenomenon and to test and validate physical law; and (3) to illustrate the role that experiment plays in relation to theory in the physical sciences.

The student should acquire knowledge about and understanding of ...[under construction]

Specifically, after completion of the course, the student should be able to:

- Show the way science progresses from observation and classification of phenomena through model building to the development of comprehensive theories that can explain and predict and that can be tested by experiment.
- Discuss the criteria for a successful scientific theory and apply those criteria to the real world.
- Apply the methods and procedures of science through laboratory exercises and observation. Analyze simple experiments and discuss whether they support or confront a theoretical prediction.
- Perform an error analysis on an experiment and draw correct conclusions from the analysis regarding the success or failure of the experiment.

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Teaching Methodologies/Activities

Physics is taught as a combination of lectures and laboratory work. In the classroom the instructional method is a traditional lecture supplemented with some audio/visual materials.

In the laboratory the environment is a guided tutorial. Students will be grouped into teams of two or three which work collaboratively to perform the experiments. Each experiment will be introduced by the instructor who will then give individualized help to each team or student. The mathematics used will be individually reviewed, demonstrated, and taught if necessary. Personal assistance is always available.

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Laboratory Rules and Procedures

Grades

Laboratory grades will be based on the written laboratory reports and on the general quality of the student's performance in the laboratory. Extra effort in lab will result in a better grade. Lab quizzes and a lab final may be given at the discretion of the instructor. (Especially if students are arriving unprepared and are not reading this Guide or the Laboratory Manual.)

All assigned experiments must be performed in the laboratory and a report submitted in order for a student to receive a passing grade. At the sole discretion of the instructor, the lowest lab grade or one missed lab may be forgiven at the end of the semester. In this event a lower grade may result.

In unusual circumstances and at the discretion the instructor, a student who has performed all the scheduled experiments but has not completed the laboratory reports may request a grade of "I" from his or her instructor. If one or more scheduled experiments have not been performed then the instructor must have permission from the course director before an "I" may be awarded because completion will require performing makeup laboratories after the close of the semester and thus requires special permission.

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Conduct

Students are expected to arrive on time for laboratory. The first part of every lab will consist of an orientation to the experiment, the equipment, and any special procedures to be followed. This is very important information and must not be missed. This is also when any safety issues will be discussed. Cell phones and pagers must be turned off in all classes: lab or lecture. For further information, refer to the University Attendance Policy.

Students will generally work in teams of two. Each member of a team is expected to participate fully in performing each experiment. Part of the data taken should always be the name of your lab partner. Do not transfer equipment from one station to another. The equipment is in matched sets which must be maintained.

Do not attempt to repair or modify a piece of equipment. If it appears that a piece of apparatus is defective or malfunctioning, report it to your instructor. Do not touch equipment that is not part of your experimental setup.

Be prepared. Read the laboratory manual before you arrive to perform a particular experiment. Lack of preparation will result in poor experimental data, lab quizzes and lower grades. Record your data in a neat planned manner. A carefully labeled table is usually best.

Do not leave the laboratory until checked out by your instructor. He or she will check your station before initialing your data sheet. The initialed data sheet indicates that you performed the experiment and properly replaced all equipment. The initials do not imply approval of the actual data. The initialed data sheet must be handed in with the lab report as an appendix.

Generally, students are expected to remain in the laboratory for the entire period. If a particular experiment does not require the entire period then calculations should be begun to ensure that the data is valid and the method(s) of analysis understood. A great deal of grief can be avoided by finding problems such as failure to make a measurement before leaving the laboratory.

Students who are well prepared and work rapidly should also be prepared to go beyond the minimum specified in the manual. All experiments have places where the minimum procedure can be expanded upon by increasing the quantity or quality of data or by taking additional measurements. Students who perform the minimum procedure by taking the minimum data in the minimum time should expect to receive a minimum grade; very good or excellent work involves performance beyond the minimum.

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Equipment to Bring to Lab

Students are expected to bring a pen, pencil, scientific calculator, and their laboratory manual with them to each laboratory session. Calculators for use in science labs should, at a minimum, be able to handle scientific notation for numbers using exponents and be able to compute the elementary transcendental functions such as sin(x), cos(x), and e^x. The ability to compute the average and standard deviation of a set of numbers is very useful as is the ability to perform a linear regression. A Texas Instruments TI-89 is an excellent choice. Each student is expected to provide his or her own graph paper. Specialty paper such as semi-log paper will be provided when needed.

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Attendance and Makeup Procedures

Attendance in laboratory is required. Completion of all the scheduled experiments in the laboratory is also required. A laboratory that is missed due to illness or other unavoidable circumstance may be made up in another laboratory section. If this is necessary the following shall apply:

- The laboratory must be made up no later than the end of the week immediately following the normal week scheduled for the lab. The equipment will not be available in the room after this time. Missed labs not completed within this period must be scheduled with the course director.

- A student wishing to make up a laboratory shall present him or herself at the start of another regularly scheduled section of the laboratory. The schedule is posted on the door of the room where the laboratory is taught. The instructor will assist the student in making up the laboratory. Since the instructor's primary obligation is to the regularly scheduled students, it is critical that the visitor be well prepared.
- Upon completion of the laboratory it is mandatory that the instructor with whom the makeup lab was performed sign and date the student's data sheet. If this is not done, the lab report will not be accepted by the student's instructor.
- No more than one experiment may be made up in a given week. For this purpose, multi-week experiments shall count as multiple experiments. Students wishing waiver of this or any other regulation must have written permission of the course director.

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Recording Data

When recording answers or data, be neat and organized. List data in tabular form. Label the data table with enough information to clearly tell what the numbers mean. If you do not do this, you will not be able to remember what your data means when you are ready to write your report. Specifically, list the instrument being read, the part of the experiment being performed, the quantity being measured, the units used and any other relevant information. A schematic diagram of the experimental set-up showing the quantities being measured is often helpful. Record data in ink. Do not erase or obliterate incorrect readings. Cross them out neatly and legibly with a note as to the problem if relevant. Never make any calculations prior to recording a reading regardless of how trivial. Record the number you see on the instrument or apparatus. Do not change units or translate the reading in any way. The original data sheets, signed by the instructor, must be handed in with the lab report. They may rewritten and reorganized for the lab rort, but the originals must be handed in.

Be careful with significant figures. Do not drop trailing zeros if they are significant. When you read an analog instrument, always estimate to one tenth of the smallest division on the device. Record all the digits from a digital instrument. Retain enough digits in intermediate calculations that you do not jeopardize your accuracy. Beware of round-off error when using calculators in their fixed decimal display mode. Do not retain digits that have no meaning. Your calculator may have trig functions accurate to ten significant figures but no reading you will ever take in a laboratory will have this accuracy. Always be aware of how many digits are significant. For each type of calculation made from your data, show one sample calculation in detail; include the proper units. Do not show all your arithmetic.

All physical quantities have an uncertainty associated with them. Significant figures are no more than a shorthand method of keeping track of uncertainty. The proper method of determining uncertainties for this course will be discussed by your instructor. However, you should always state the uncertainty of your results; no result is complete without a statement of its uncertainty. The conclusion of a report must contain a discussion of the relation between your result, its uncertainty, the accepted value and its uncertainty. A proper analysis of error is an important part of any lab report.

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Graphing Data

Graphs are among the most important methods of presenting and analyzing experimental data. A good graphical presentation of data will allow patterns (and problems) to be seen easily. Good graphs often form the heart of a laboratory report. Graphs are drawn to enable conclusions concerning the physical law involved in the experiment. Hence no graphs is complete unless it is in a form in which it can be interpreted and unless the proper conclusions are drawn.

When the points of a graph lie on a straight line inclined to the axis, a linear relationship exists between the variables. The equation for such a line is $y=mx+b$ where y is the dependent variable, x the independent variable, m the slope of the line, and b the y-intercept. Linear relations are easy to graph and easy to recognize. Many relations are non-linear. It is quite difficult to recognize whether a particular data set has a specific functional dependance. In these cases it is common to introduce an auxiliary variable so that the data is linear in the new variable

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Types of Graphs

Several types of coordinate systems are in general use - Cartesian or rectangular, polar and logarithmic coordinates are among these. The rectangular coordinate system is most commonly used in elementary physics and is the only one considered here. There are several rules to be observed when plotting graphs using this system.

A smooth curve of best fit should be drawn through or near the experimental points. Never connect the data points with line segments. Marker pens make poor graphs because they draw fat lines. Use a fine line for graphs. Each graph should be discussed in the body of the report.

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Titles and Axes

Each graph must have an informative title. For example "Pressure of Saturated Water Vapor vs Temperature" is more informative than a cryptic "P vs T". For a graph drawn by hand, the title and axes should be in ink while the data points and the curve should be in pencil.

Draw two mutually perpendicular lines to represent the coordinate axes. On each of these mark the name of the quantity to be plotted. The horizontal axis is used for the independent variable (the variable whose value was controlled or chosen in the experiment) and the vertical axis for the dependent variable (the variable which was measured and whose value depends on the independent variable).

The axes of each graph must be clearly labeled stating what is plotted and in what units. The scale must be metric and use decimal fractions. The scales used on the two axes should be chosen so that the graph fills the page. The axes normally should include the origin and should be chosen so that the graph is not squeezed onto one edge of the paper or into one small square.

Whenever possible, the scale should be such that the data can be read back from the curve to the proper number of significant figures. This usually means a graph the full size of the paper when the data contains 3 significant figures. The scale should be convenient to read, that is, it should be easy to interpolate between the scale markings in order to find the proper place for recording the points on the graph. Indicate the proper units on each axis. It is usually advisable to have the origin appear on the paper. However, if one of the variables has only a small range of values and this range is far from zero, it may not be desirable to include the point (0,0) on the graph. The scales on both axes need not be the same.

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Data Points and Curves

Each plotted point should be made visible either by drawing a small circle or other shape around it or a cross through it. If appropriate, draw a straight line or a smooth curve which best represents all the points. This is a line or curve which is as close as possible to most of the points and which has approximately as many points on one side of it as on the other. Any point which deviates widely from this curve is likely to be in error.

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Computer Graphing Programs

There are many computer software systems available that will produce very good graphs of experimental data. Most will also do some curve fitting such as a linear regression. The commonest is Microsoft's Excel. Excel has the advantage of being widely available and its graphs can be easily pasted into Word or WordPerfect documents. Its disadvantage is that its graphing engine is designed primarily for business not scientific use.

Other commercial systems include MathCad, Axum, MatLab, Mathematica, and Maple. These are all powerful systems designed for scientific or engineering use. They can also be quite expensive. There are also several very powerful open source (free) systems available. Python is being used extensively in the scientific community for both data analysis and graphical presentations of data. It has several graphing packages available, Matplotlib being one of the most accessible for 2D graphs.

When using graphical software, especially Excel, there are several things to watch out for. Make sure your graphs are big enough. Excel tends to create small graphs. Take the time to add all the titles and labels that a good graph must have. Beware of the default settings, they are often designed for business not scientific use. Do not allow the software to connect the dots. Learn to add a straight line to your graph.

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Terminology

It is necessary to know the definitions of certain terms used in the rectangular system of coordinates in order to be able to discuss the graph when it is plotted. The most important of these definitions follows:

Origin of coordinates: The point the vertical and horizontal axes cross.

Intercept: The distance from the origin to the point where the graph crosses the axis.

Slope of a line: The ratio of the vertical difference between the coordinates of two points to the horizontal difference between the coordinates of the same. When the graph is not a straight line the slope of the curve at any point is the slope of the line tangent to the curve at that point.

Linear Regression or Least Squares Fit: The most important mathematical technique for fitting a curve (usually a straight line) to a set of data.

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Laboratory Reports

Lab Report Rules

Lab reports must printed with the original signed data appended. They must be neat, clear and well organized. Each report is to be printed on 8.5" x 11.0" paper. The cover page of the report should contain the name of the report, your name, the date it was performed, the date it was submitted and the names of your lab partners.

Lab reports are normally due one week after completion of the experiment. Late reports will have credit deducted. Reports may be rewritten and handed in again. They will be re-graded and the highest grade used. The only condition is that the original report must be submitted along with the re-write.

Fairleigh Dickinson University has an Academic Integrity Policy that each student must read and understand. It also has a formal procedure for appealing a grade. Both documents can be found in the *Student Handbook* and on the FDU web site. Students should be aware that material downloaded from the Internet is subject to the same conditions as material copied from any other source. Lab reports must be based on your own data, taken by you in collaboration with your partners. If you did not participate in the taking of the data, use of another's data is plagiarism just as use of another's words is plagiarism

In the laboratory, students will generally work collaboratively in teams of two or three. Each member of a team is expected to participate fully in performing each experiment. Collaboration in understanding and analyzing the results of an experiment is expected. You should always include the names of your lab partners as part of each report. However, laboratory reports, like any other written work, must be original and your own. The raw data you use will usually be identical to that of your partners; your analysis and the words used to discuss and analyze it must be independent. Each member of a team must write their own lab report. Your report must be based on your own data, taken by you in collaboration with your partners. Use of another's data without attribution is plagiarism just as use of another's words is plagiarism.

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Types of Laboratory Reports

In the sciences, laboratory reports generally come in two formats: a *long form* report and a *short form* report.

Long form reports are written for a reader who is totally unfamiliar with the particular experiment being reported on or why it was performed. They contain enough material that a reader unfamiliar with the subject and the laboratory could reconstruct the entire experiment. Long form reports normally include a discussion of the theoretical background and context of the experiment, a complete discussion of the procedures and protocols that were followed, and a full description of the equipment used. The results are then presented and analyzed and appropriate conclusions drawn. These reports are often quite lengthy.

Short form reports are written assuming that the reader is familiar with the theoretical background of the experiment, the course text, the laboratory manual and the procedures the manual specifies, and with the equipment provided in the lab. Short form reports omit several of the background discussions present in a long form report. They do not include the theory. It is assumed that either the text or the laboratory manual have good discussions of the theory. They do not repeat the procedures or the equipment descriptions that are present in the laboratory manual. It is assumed that the reader is familiar with what the student was supposed to do. However, it is not assumed that the experimenter actually did what was specified. A short form report must contain enough discussion of the procedures followed so that the reader can be assured that the experimenter understood and actually did what was specified by the laboratory procedures or protocols. Deviations from or alterations to the procedures are always recorded.

A long form report assumes the reader was unfamiliar with the experiment and its background. A short form report assumes that the reader knows what was expected of the student including the procedures to be followed, the equipment provided, and that background of the experiment. It does not assume that the experiment was performed correctly and was understood and worked. That must be shown in the report. In *Physics* we will write short form laboratory reports.

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General Report Requirements

A laboratory report is an example of technical writing in a scientific context. There are several features that all college level technical writing has in common. The report must be in grammatically correct English with proper spelling, capitalization, and punctuation. A laboratory report is a *report*. It is not a journal, a diary, or a “blog”. It should be written in a moderately formal style, carefully organized, and carefully presented. It should be formatted in a manner that aids the reader in seeing and understanding its content. Paragraph breaks and indentation should be used freely to organize the material visually.

Data and calculations should be presented in tables or columns, never embedded in running text. Numbers should always have proper units and be carefully labeled as to what quantity they represent. Differentiate between the original data and your calculations or analysis. While a good presentation often requires data to be reorganized from the original format in which it was recorded, the original data must always be included in a report. Relegate it to an appendix if it is truly ugly and would spoil the appearance of an otherwise will organized and carefully presented report. However, *the original data, signed by the instructor, must be included with the report.*

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Laboratory Report Organization

Lab reports must printed with the original signed data appended. They must be neat, clear and well organized. Each report is to be printed on 8.5" x 11.0" paper. The names of your lab partners must be clearly listed.

Laboratory reports should generally have a cover page and three main parts: (1) Introduction, (2) Data Analysis and Discussion and (3) Conclusion. The cover page should include the title of the experiment, the date(s) it was performed, your name, the name(s) of your lab partner(s), and the date it was submitted.

> (1) The *Introduction* should contain a brief statement of the purpose or objective of the experiment as you understand it. What is being tested or demonstrated? What laws are involved? What should the outcome be? Generally, a single paragraph will suffice.
>
> (2) The *Data Analysis and Discussion* is the heart of the laboratory report. Each answer, calculated result, and graph should be presented and its significance discussed. The discussion should relate to the objective stated in the introduction. Indicate the reliability of each result by the number of significant figures you list. Discuss sources of error and evaluate which are significant. Note that different experiments differ widely in the accuracy obtainable. The actual size of the error, 1% or 20%, is less important than a correct assessment and discussion of the result and outcome of the experiment. Your ability to take good data is important but your ability to draw correct conclusions from whatever data you took is more important. Always include your original initialed data sheet with your report, even if you copy and re-present the data in your report.
>
> No lab report is complete without a careful analysis of the data and the errors or uncertainties involved.
>
> (3) The *Conclusion* should be a brief paragraph of hindsight. This is where you bring together the rest of the report into a few paragraphs stating the final result, success or failure, of the experiment. This is where your final numerical results, with the proper number of significant figures and proper units, are summarized. It is vitally important that your conclusion be a valid reflection of the actual outcome you achieved. One of the worst transgressions in science is to misrepresent the outcome of an experiment.

The report should not contain an extensive discussion of theory. Your text and the laboratory manual contain enough theory. In addition, procedure should be discussed in a manner that supplements the manual rather than duplicates it. See the section on *Types of Laboratory Reports* above. Also, the report must be your own work – plagiarism will not be tolerated. See the Physics Lab I Syllabus or the Physics Lab II Syllabus for details.

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Error Analysis[1]

Introduction to Experimental Uncertainty and Errors

Physics is a science which includes a quantitative study of natural phenomena and the relationships which exist among these phenomena. Therefore, a knowledge of the methods by which data establishing these relationships are collated is of fundamental importance.

In courses in mathematics, when problems involving addition, subtraction, multiplication and division are solved, the numbers are usually considered exact and no question concerning their accuracy arises. But measurements of physical quantities are never exact because of the limitations of the measuring instrument or of the method used and because the physical quantities themselves may not have exact values. For this reason errors in data are inevitable. At the conclusion of each experiment it is essential to have some idea of the reliability of the result and to state the result in a way which will indicate this reliability. It is, therefore, necessary to study the nature of experimental errors and become familiar with a conventional way of recording a result.

Every number you measure in the laboratory has some uncertainty or error associated with it. These errors produce uncertainties in the calculations and ultimately in the final value for quantities you calculate and then wish to compare to theoretical or handbook values. In order to assess the success or failure of an experiment it is essential to know the errors present. We will return to this point below. In what follows we will give a very brief discussion of the major points of error analysis. For further discussion and for proofs of those statements we make the reader is referred to the appropriate sections of Taylor.

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How to Report and Use Uncertainties

The correct way to report the result of a measurement is to give a [best estimate] ± [uncertainty] or, in equation form,

$$(\text{measured value of } x) = x_{\text{best}} \pm \delta x$$

where x_{best} is your best estimate for the value and δx is the uncertainty. What you are saying by giving an uncertainty is that you have some degree of confidence that the actual value lies between $x_{\text{best}}-\delta x$ and $x_{\text{best}}+\delta x$. How to get your "best estimate" and the associated "uncertainty" and exactly

[1] The discussion in this section is based on *An Introduction to Error Analysis: The Study of Uncertainties in Physics Measurements, 2nd Ed.*, by John R. Taylor.

what we mean by “some degree of confidence” is what the general subject of “error analysis” is all about. There are several levels of sophistication that can be achieved. We will stay at the elementary level in this discussion.

There are two basic rules associated with uncertainties and best values. First, there is absolutely no purpose served by quoting an uncertainty to high precision. The rule for normal work is to use 1 or at most 2 significant figures. *Do not quote an uncertainty to more than 10% accuracy.* Second, do not quote your best estimate more (or less) accurately than the associated uncertainty. *The best value and its uncertainty should have their least significant figure at the same decimal place.*

In the preceding paragraph we have used the term “significant figure” several times. Significant figures are a simplified way to represent uncertainty. The scheme has its origin in the definition of the word “significant” which means “having meaning”. Significant figures must all “have meaning” given the actual uncertainties. Saying that a measured value is 5.1762 with an uncertainty of 0.02 clearly means that the final two digits “62" are meaningless – not significant.

The words "precision" and "accuracy" when used in the discussion of measurements have quite different meanings. All blunders must be eliminated before either term can be applied. The precision can be increased by reducing the random errors. This can be done by taking more readings or by increasing the refinement of the instrument. An increase in the accuracy of a measurement implies both an increase in precision and the reduction of any systematic errors which may be present in the instrument or method.

d) Using Errors: A correctly done error analysis is an essential part of a properly written lab report. It can indicate the quality of your data and is essential in deciding whether or not the experiment was satisfactory. As an example, suppose that you have measured the sum of the torques on a rigid body and found that it is 450 dyne-cm. Without an error statement this result cannot be evaluated. If the errors had come to ±600 dyne-cm, then the sum could have been anywhere between +1,000 dyne-cm and -200 dyne-cm which is consistent with zero. If, however, the errors were ±50 dyne-cm then the sum could have been between 400 dyne-cm and 500 dyne-cm which is not consistent with zero and you would have to conclude either that the second condition of equilibrium is wrong or (more likely) that systematic errors were present.

To generalize this example, if $A \pm \delta A$ is the outcome of your experiment and A_{theor} is the theoretical value then the experiment was a success if

$$A - \delta A < A_{theor} < A + \delta A.$$

we need to define a second method of stating the error in a quantity. The error δA in a quantity A as discussed above is a positive number with the same units and dimensions as A. δA is often referred to as the absolute error in A. Another useful quantity is the relative error, $\delta A/A$, or the percent error, %A.

$$\%A \equiv \delta A \times 100 / A$$

Relative error or percent is a ratio and thus a pure number without units. To avoid confusion with other numbers we shall always use percent error. The reader should note that percent and relative error differ by a constant numerical factor of 100 that we may occasionally omit. Conversion from percent error back to absolute error is easily performed using

$$\delta A = \%A \cdot A / 100$$

or, in words, δA is "%A percent of A".

We close this discussion on error with a few comments on the proper way to quote errors. Absolute errors are normally quoted with only one significant figure although if the error is between 1 and 3 two figures are sometimes given. The error is thus quoted to about ten percent accuracy. The measured or calculated value is never given more accurately than the error. This is particularly important in these times of calculators that disgorge ten digit answers at the touch of a 1/X key. (It should also be noted that a number is never given less accurately than its error.) A few examples are given below.

Incorrect	Correct
3.13 ± 0.4	3.1 ± 0.4
752 ± 39	750 ± 40
754.79 ± 1.83	754.8 ± 1.8
3 ± 0.01	3.00 ± 0.01
2.79315 ± 0.02	2.79 ± 0.02

ETC.

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How to Estimate Errors

a) Classifying Errors. The errors associated with measurements are generally divided into two groups, random and systematic. Random errors are equally likely to be too large or too small and are amenable to statistical analysis. Systematic errors are always either too large or too small and cannot be analyzed statistically. Both types of error can be reduced by improved experimental design, and technique and by improved instrumentation. If a source of systematic error is suspected its effects can often be estimated. In what follows we shall be concerned with handling random error and estimating its effect. In the literature a third type of error is often discussed. This is called the "blunder" and is self explanatory.

There are three classes of errors which can occur:

(1) Mistakes or Blunders. Mistakes on the part of the observer in reading instruments, recording data and calculating results are completely unpredictable. These errors can be eliminated by being careful. Repeating a measurement is often a help in detecting a blunder, although it is usually better to have a second observer independently check the reading.

(2) Systematic Errors. Systematic errors can be introduced by the use of an instrument which has a fixed error, or by an observer who consistently over or underestimates a result in a measurement where human judgment is necessary. Thus, all data will have the same error. If a scale which is correct at one temperature is used at a higher one, the expansion of the scale will lead to length measurements which are always too small and the relative error will be the same no matter what the length measured. For example, if the error with this scale in measuring a distance of 100 ft. is 0.1 ft., the error in measuring a distance of 200 ft. will be 0.2 ft. If an observer, in reading fractional parts of a scale division, always estimates 0.2 ft. too high, the same absolute error will occur in all his measurements, but the relative error will be greater the smaller the distance measured.

(3) Random Errors. Small accidental errors are present in all measurements because of the limited refinement of the measuring instrument or for unknown or indeterminate reasons. An example of the former is the use of a scale whose smallest sub-division is 0.1 cm. Random errors will occur when this scale is used to estimate a length to 0.01 cm. An example of the latter would be the small random variations in the size of scale divisions. Positive and negative random errors are equally probable. Since these errors in general follow the laws of chance, their effect can be predicted mathematically.

b) Estimating Errors. There are three methods for estimating the errors in your original measurements. The first and best method is to make repeated measurements of the quantity, find the average value and then to use either the average or the standard deviation as the error.

Although it is the method of choice, finding the average deviation is not always the best way to estimate the error: sometimes time constraints make it impractical to take ten (or even five) measurements of the same quantity; sometimes the quantity cannot be measured more than once because the initial conditions of the experiment cannot be reproduced; and sometimes the average deviation is an unrealistically small estimate of the error. When any of these occur, two other methods are available to estimate the error. The first of the two alternate methods is simply to estimate the scale accuracy of the measurement. For example: a meter stick cannot be read to better than ~ 1/3 mm; a mercury-in-glass thermometer has an error of at least ±.1°K and a dial thermometer may have an error of several degrees K; digital devices are always uncertain by ±1 in the last digit in addition to their general accuracy; and finally your reaction time is a few tenths of a second and some timers are worse. As a rule-of-thumb, if the average deviation is much smaller than the scale accuracy of the instrument, the scale accuracy should be quoted as the error. (This rule is modified in those cases where division by $\sqrt{N}$ is appropriate).

The last method for estimating errors is to find how sensitive the experiment is to a small change in one of the parameters. For instance, suppose an experiment involves hanging a mass on a string. In principle the mass, which might be 100 gms, could be measured to within a few milligrams, giving a very small error. However, it may be that the mass could be increased by as much as 5 gms without producing any detectable change in the experimental set up. In this case, since the 100 gm mass could have been replaced by a 105 gm mass without affecting the experiment, the error is 5 gms. This method is a short cut that avoids making repeated measurements by estimating the speed of values that would occur if repeated measurements were made.

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Propagation of Errors

c) Propagation of Errors: After the original measurements have been made, all experiments involve some calculations to arrive at a final quantity that can be compared to the theory. The errors in the measured quantities produce errors in the calculated ones. This process is called propagation of error. The general analysis of error propagation is quite complex and the interested reader is again referred to Taylor or to more advanced treatments. We will give a simplified treatment that will allow you to estimate the errors in a calculated quantity with a minimum of effort.

Before we discuss error propagation we need to define a second method of stating the error in a quantity. The error δA in a quantity A as discussed above is a positive number with the same units and dimensions as A. δA is often referred to as the absolute error in A. Another useful quantity is the relative error, $\delta A/A$, or the percent error, %A.

$$\%A \equiv \delta A \times 100 / A$$

Relative error or percent is a ratio and thus a pure number without units. To avoid confusion with other numbers we shall always use percent error. The reader should note that percent and relative error differ by a constant numerical factor of 100 that we may occasionally omit. Conversion from percent error back to absolute error is easily performed using

$$\delta A = \%A \cdot A / 100$$

or, in words, δA is "%A percent of A".

Returning to propagation of error, let us consider the error in a sum. Let $c = a + b$ where $a \pm \delta a$ and $b \pm \delta b$ are experimental values. We can write

$$c \pm \delta c = (a \pm \delta a) + (b \pm \delta b)$$

and assuming the worst case

$$c + \delta c = (a + b) \pm (\delta a + \delta b)$$

giving

$$\delta c = \delta a + \delta b$$

Since c could have been negative this proof works equally well for subtraction. We summarize by stating:

> The absolute error of a sum or difference is the sum of the absolute error.

To extend this to multiplication assume $A = AB$ and consider

$$C \pm \delta C = (A \pm \delta A)\,(B \pm \delta B)$$

$$= AB \pm B\delta A \pm A\delta B + \delta A\delta B$$

Noting that $C = AB$ and dropping the term $\delta A\delta B$ as very small we find

$$\delta C = B\delta A + A\delta B$$

Dividing both sides by $C = AB$ gives

$$\delta C/C = \delta A/A + \delta B/B$$

or

$$\%C = \%A + \%B.$$

This can be generalized as

> The percent error in a <u>product</u> is the sum of the percent errors.

This rule also applies to division because the relative error in 1/C is the same as the relative error in C, if the error is small. To see this let $C' = 1/C$ where C has absolute error δC.

$$C' \pm \delta C' = [C \pm \delta C]^{-1}$$

$$= C^{-1}\cdot[1 \pm \delta C/C]^{-1}$$

$$\approx C^{-1} \cdot [1 - (\pm\delta C/C)]$$

$$= C' \cdot [1 \pm \delta C/C]$$

Dividing both sides by C' gives

$$1 \pm \delta C'/C' = 1 \pm \delta C/C$$

and therefore

$$\%C' = \%C$$

These two simple rules will handle almost all the situations that you will encounter this year.

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Statistical Analysis of Random Errors

b) Estimating Errors. There are three methods for estimating the errors in your original measurements. The first and best method is to make repeated measurements of the quantity, find the average value and then to use either the average or the standard deviation as the error. We now discuss this method in detail.

Suppose you measure a quantity *A* N times resulting in values A_i, i = 1,...,N. The average, <*A*> or $\overline{A}$, is defined as

$$\overline{A} \equiv \left(\sum_{i=1}^{N} A_i \right) \Big/ N$$

and the individual deviation of each measurement is

$$\delta A_i \equiv A_i - \langle A \rangle$$

The average <A> is the best estimate of the quantity and is the value to be used in subsequent calculations. The error, δA, in <A> can be found as the average deviation

$$\delta A = \text{avg. dev.} \equiv \Sigma\, |\delta A_i|/N$$

or as the standard deviation

$$\delta A = \sigma \equiv \sqrt{}\, \Sigma\, (\delta A_i)^2/(N-1)$$

Note the absolute value sign in the definition of the average deviation. When calculations must be done by hand or with a calculator the average deviation is used. However, some calculators have a built in key for calculating the standard deviation. If such a calculator is available, then the standard deviation should be used as it is mathematically preferable. Under certain circumstances the error quoted should be $\delta A/\sqrt{N}$ rather than δA. Refer to a text on error analysis to find when this is allowed. This average deviation method should always be used with N at least five and preferably ten.

In conclusion, no measurement is complete without an estimate of its uncertainty or error. Each experimental quantity should be given in the form

$$A \pm \delta A$$

where δA, a positive number with units, is the (absolute) error determined by one of the methods discussed above.

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